Control design for distributed parameter systems : The port Hamiltonian approach.

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Thank you for your attention !

Conclusions and future works

Conclusion

- We provided an overview on some key results on control of distributed port Hamiltonian systems in the 1D case.
- We detailed a constructive control design technique : energy shaping for boundary/in domain controlled DPS.
- We proposed first ideas on observer design.
- We presented some possibles extensions to irreversible thermodynamic systems.

Future works

- Study of the impact of the distribution of the patches on the achievable performances.
- Control design for a class of non linear PDE systems.
- Extension to 2D DPS.
- Control design for irreversible PHS.

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Outline

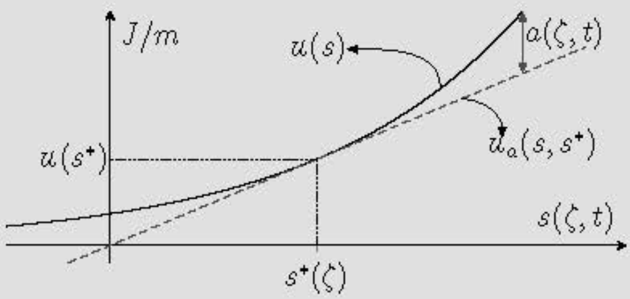
1. Context and motivation
2. Infinite dimensional Port Hamiltonian systems (PHS)
3. Control by interconnection and energy shaping
4. Irreversible boundary controlled port Hamiltonian Systems
5. Conclusions and future works

Control design

Idea

- Use the Thermodynamic availability function as closed loop Lyapunov function.

$$\mathcal{A} = \int_0^L (u(s) - u_a(s))\, dz$$



- Use Entropy Assignment to guarantee the convergence of trajectories.

It has been successfully applied to the control of the heat equation. More complex systems (reaction-convection-diffusion systems) are under investigation.

The non-isentropic fluid : the irreversible case

The system of balance equations may be written as the quasi-Hamiltonian system

$$\begin{bmatrix} \frac{\partial \phi}{\partial t} \\ \frac{\partial \upsilon}{\partial t} \\ \frac{\partial s}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial (\cdot)}{\partial z} & 0 \\ \frac{\partial (\cdot)}{\partial z} & 0 & \frac{\partial}{\partial z}\left(\frac{\hat{\mu}}{T}\left(\frac{\partial \upsilon}{\partial z}\right)(\cdot)\right) \\ 0 & \frac{\hat{\mu}}{T}\left(\frac{\partial \upsilon}{\partial z}\right)\frac{\partial (\cdot)}{\partial z} & 0 \end{bmatrix} \left(\begin{bmatrix} \frac{\delta H}{\delta \phi} \\ \frac{\delta H}{\delta \upsilon} \\ \frac{\delta H}{\delta s} \end{bmatrix} \right)$$

From this new formulation (skew symmetry of the differential operator) one can define the energy/entropy boundary port variables (and input/output) such that :

$$\frac{dH}{dt} = y^T \nu$$

and

$$\frac{dS}{dt} = \underbrace{\int_a^b \sigma \, dz}_{\geq 0} + y_s^T \nu_s$$

The non-isentropic fluid : the irreversible case

We can account for the thermal domain by considering Gibbs' equation

$$du = -pd\phi + Tds$$

where s denotes the entropy density and T the temperature. The total energy of the system is still the sum of the kinetic and the internal energy but now depends on s

$$H(\upsilon, \phi, s) = \int_a^b \left(\frac{1}{2}\upsilon^2 + u(\phi, s)\right) dz$$

From the conservation of the total energy and Gibbs' equation $\frac{\partial u}{\partial s} = T$ we get

$$\frac{\partial s}{\partial t}(t,z) = \frac{\hat{\mu}}{T}\left(\frac{\partial \upsilon}{\partial z}\right)^2 (t,z)$$

Irreversible systems

We consider a 1-D isentropic fluid in Lagrangian coordinates, also known as *p-system*, with $[a, b] \ni z$, $a, b \in \mathbb{R}$, $a < b$. We choose as state variables

- the specific volume $\phi(t, z)$,
- the velocity $\upsilon(t, z)$ of the fluid.

System of two conservation laws :

$$\frac{\partial \phi}{\partial t}(t,z) = \frac{\partial \upsilon}{\partial z}(t,z)$$
$$\frac{\partial \upsilon}{\partial t}(t,z) = -\frac{\partial p}{\partial z}(t,z) - \frac{\partial \tau}{\partial z}(t,z)$$

where $p(\phi)$ is the pressure of the fluid, $\tau = -\hat{\mu}\frac{\partial \upsilon}{\partial z}$ with $\hat{\mu}$ the viscous damping coefficient. The total energy of the system is given by the sum of the kinetic energy and internal energy :

$$H(\upsilon, \phi) = \int_a^b \left(\frac{1}{2}\upsilon^2 + u(\phi) \right) dz$$

$$\begin{bmatrix} \frac{\partial \phi}{\partial t} \\ \frac{\partial \upsilon}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{\partial}{\partial z} \left(\begin{bmatrix} \frac{\delta H}{\delta \phi} \\ \frac{\delta H}{\delta \upsilon} \end{bmatrix} \right) + \begin{bmatrix} 0 & 0 \\ 0 & \frac{\partial}{\partial z}\left(\hat{\mu}\frac{\partial \cdot}{\partial z}\right) \end{bmatrix} \left(\begin{bmatrix} \frac{\delta H}{\delta \phi} \\ \frac{\delta H}{\delta \upsilon} \end{bmatrix} \right),$$

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Control by interconnection (Achievable performances)

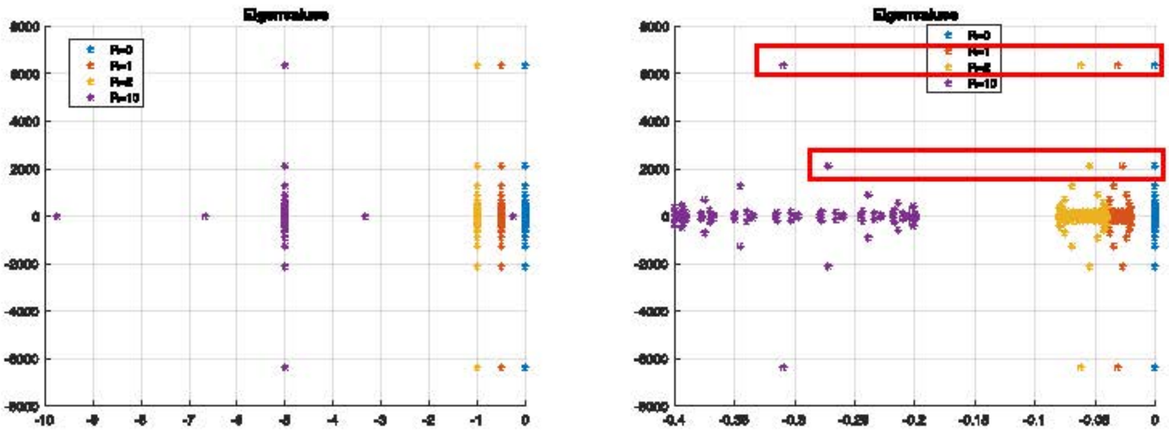


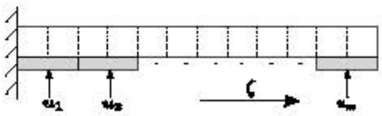
FIGURE – Control by interconnection. Full actuation (left), partial actuation (right).

Application case (2) (energy shaping +damping injection)

Application case (2) (damping injection)

Energy shaping : application (1)

We consider the control of a weakly damped Timoshenko beau using 50 homogeneously distributed patches.



Stability analysis

The controller is now connected to the infinite dimensional system leading to :

$$\dot{\mathcal{X}} = \underbrace{\begin{pmatrix} (\mathcal{J} - \mathcal{R} - \mathcal{B}D_c\mathcal{B}^*) & -\mathcal{B}B_c^T \\ B_c\mathcal{B}^* & 0 \end{pmatrix} \begin{pmatrix} \mathcal{H} & 0 \\ 0 & Q_c \end{pmatrix}}_{\mathcal{A}_{cl}} \mathcal{X}, \tag{44}$$

where $\mathcal{X} = \begin{pmatrix} x^T & x_c^T \end{pmatrix}^T \in X_s$ where $X_s = L_2\left([0, L], \mathbb{R}^{2p}\right) \times \mathbb{R}^m$.

Existence of solution, stability analysis

- The operator $\mathcal{A}_{cl}$ defined in (44) generates a contraction semigroup on $X_s = L_2\left([0, L], \mathbb{R}^{2p}\right) \times \mathbb{R}^m$.
- The operator $\mathcal{A}_{cl}$ has a compact resolvent.
- Asymptotic stability : For any $\mathcal{X}(0) \in L_2\left([0, L], \mathbb{R}^{2n}\right) \times \mathbb{R}^m$, the unique solution of (44) tends to zero asymptotically, and the closed loop system (44) is globally asymptotically stable.

Energy shaping

Approximate energy shaping [Liu et al., 2021]

Choosing $J_c = 0$, and $R_c = 0$, the closed loop system (38) admits :

$$C(x_{1d}, x_c) = B_c M^T B_{0d}^T J_i^{-1} x_{1d} - x_c \tag{41}$$

as structural invariant along the closed loop trajectories. The control law (37) is a PI action equivalent to the state feedback :

$$\mathbf{u}_d = -B_c^T Q_c B_c M^T B_{0d}^T J_i^{-1} x_{1d} - D_c M^T B_{0d}^T Q_2 x_{2d}. \tag{42}$$

Therefore, the closed loop system yields :

$$\begin{pmatrix} \dot{x}_{1d} \\ \dot{x}_{2d} \end{pmatrix} = \begin{pmatrix} 0 & J_i \\ -J_i^T & -\left(R_d + B_{0d} M D_c M^T B_{0d}^T\right) \end{pmatrix} \begin{pmatrix} \tilde{Q}_1 x_{1d} \\ Q_2 x_{2d} \end{pmatrix}, \tag{43}$$

where : $\tilde{Q}_1 = Q_1 + J_i^{-T} B_{0d} M B_c^T Q_c B_c M^T B_{0d}^T J_i^{-1}$.

$B_c^T Q_c B_c$ can be designed to minimise $\left\| \tilde{Q}_1 - Q_m \right\|_F$ (Convex optimization problem)

Control by interconnection

The closed loop system is given by

$$\dot{x}_{cl} = \left(J_{cl} - R_{cl}\right) Q_{cl} x_{cl}, \tag{38}$$

where $x_{cl} = \begin{pmatrix} x_{1d}^T, & x_{2d}^T, & x_c^T \end{pmatrix}^T$, $Q_{cl} = \text{diag}\begin{pmatrix} Q_1, & Q_2, & Q_c \end{pmatrix}$,

$$J_{cl} = \begin{pmatrix} O & J_i & 0 \\ -J_i^T & 0 & -B_{0d} M B_c^T \\ 0 & B_c M^T B_{0d}^T & J_c \end{pmatrix}, \; R_{cl} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & R_d + B_{0d} M D_c M^T B_{0d}^T & 0 \\ 0 & 0 & R_c \end{pmatrix}.$$

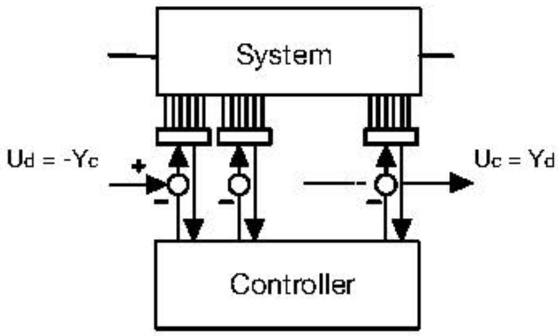
The Hamiltonian of the controller (36) is :

$$H_c(x_c) = \frac{1}{2} x_c^T Q_c x_c. \tag{39}$$

Therefore, the closed loop Hamiltonian function reads :

$$H_{cld}\left(x_{1d}, x_{2d}, x_c\right) = H_d\left(x_{1d}, x_{2d}\right) + H_c(x_c). \tag{40}$$

Control by interconnection



The controller is designed as finite dimensional PHS of the form :

$$\begin{cases} \dot{x}_c = (J_c - R_c)\, Q_c x_c + B_c u_c, \\ y_c = B_c^T Q_c x_c + D_c u_c, \end{cases} \tag{36}$$

interconnected in a power preserving way through the relation

$$\begin{pmatrix} u_d \\ u_c \end{pmatrix} = \begin{pmatrix} 0 & -M \\ M^T & 0 \end{pmatrix} \begin{pmatrix} y_d \\ y_c \end{pmatrix}, \text{ where } M = \mathbb{I}_m \otimes \mathbf{1}_{k\times 1} \in \mathbb{R}^{n\times m}, \tag{37}$$

Early lumping approach

The system is first discretized using a structure preserving method (mixed finite element method [Golo et al., 2004]) such that the approximation of (1) is again a PHS with n elements :

$$\begin{pmatrix} \dot{x}_{1d} \\ \dot{x}_{2d} \end{pmatrix} = (J_n - R_n) \begin{pmatrix} Q_1 x_{1d} \\ Q_2 x_{2d} \end{pmatrix} + B_b u_b + \begin{pmatrix} 0 \\ B_{0d} \end{pmatrix} \mathbf{u}_d, \tag{34a}$$

$$y_b = B_b^T \begin{pmatrix} Q_1 x_{1d} \\ Q_2 x_{2d} \end{pmatrix} + D_b u_b, \tag{34b}$$

$$\mathbf{y}_d = \begin{pmatrix} 0 & B_{0d}^T \end{pmatrix} \begin{pmatrix} Q_1 x_{1d} \\ Q_2 x_{2d} \end{pmatrix}, \tag{34c}$$

where $x_{id} = \begin{pmatrix} x_i^1 & \cdots & x_i^n \end{pmatrix}^T \in \mathbb{R}^{np \times 1}$ for $i \in \{1, \cdots, 2p\}$,

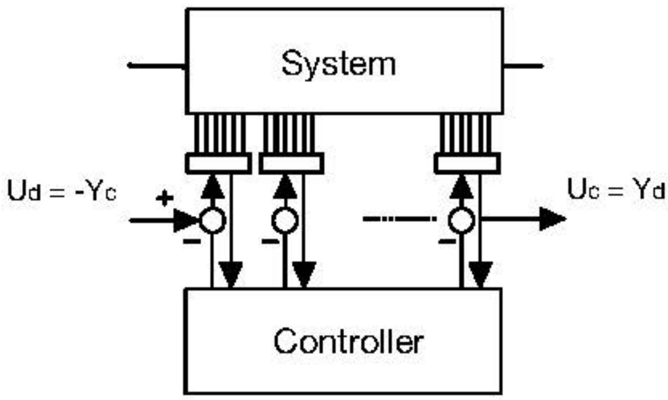
$$J_n = \begin{pmatrix} 0 & J_i \\ -J_i^T & 0 \end{pmatrix} \quad \text{and} \quad R_n = \begin{pmatrix} 0 & 0 \\ 0 & R_d \end{pmatrix},$$

The discretized energy reads :

$$H_d(x_{1d}, x_{2d}) = \frac{1}{2} \left(x_{1d}^T Q_1 x_{1d} + x_{2d}^T Q_2 x_{2d} \right). \tag{35}$$

Control by interconnection

- Non ideal case : the distributed parameter system is actuated through piecewise constant elements.



Energy shaping : ideal case

Energy shaping [Trenchant et al., 2017]

Choosing $\mathcal{B}_c = \mathcal{G}$ and $\mathcal{J}_c = 0$ the closed loop system (25) admits as structural invariants the function $C(x_e)$ defined by (26) and

$$\Psi = (\Psi_1, 0, \Psi_1)$$

In this case the hyperbolic system (1) connected to the dynamic controller (36) of the form

$$\begin{cases} \frac{\partial x_C}{\partial t}(\zeta, t) = \mathcal{G} u_C(\zeta, t) \\ y_C(\zeta, t) = \mathcal{G}^* \mathcal{Q}_c x_C(\zeta, t) + \mathcal{S}_c u_C(\zeta, t) \end{cases} \tag{31}$$

is equivalent to the system

$$\frac{\partial}{\partial t}\begin{bmatrix} x_1(\zeta, t) \\ x_2(\zeta, t) \end{bmatrix} = \begin{bmatrix} 0 & \mathcal{G} \\ -\mathcal{G}^* & -(R + \mathcal{S}_c) \end{bmatrix} \begin{bmatrix} (\mathcal{H}_1(\zeta) + \mathcal{Q}_c(\zeta))\, x_1(\zeta, t) \\ \mathcal{H}_2(\zeta) x_2(\zeta, t) \end{bmatrix} \tag{32}$$

$$u_\partial = \mathcal{B}\begin{bmatrix} (\mathcal{H}_1(\zeta) + \mathcal{Q}_c(\zeta))\, x_1(\zeta, t) \\ \mathcal{H}_2(\zeta) x_2(\zeta, t) \end{bmatrix}, \; y_\partial = \mathcal{C}\begin{bmatrix} (\mathcal{H}_1(\zeta) + \mathcal{Q}_c(\zeta))\, x_1(\zeta, t) \\ \mathcal{H}_2(\zeta) x_2(\zeta, t) \end{bmatrix} \tag{33}$$

Control by interconnection : ideal case

The closed loop system reads :

$$\frac{\partial x_e}{\partial t} = \begin{pmatrix} \frac{\partial x_1}{\partial t} \\ \frac{\partial x_2}{\partial t} \\ \frac{\partial x_c}{\partial t} \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{G} & 0 \\ -\mathcal{G}^* & -(\mathcal{S}_c + R) & -\mathcal{B}_c^* \\ 0 & \mathcal{B}_c & \mathcal{J}_c \end{pmatrix} \begin{pmatrix} \mathcal{H}_1 x_1 \\ \mathcal{H}_2 x_2 \\ \mathcal{Q}_c x_c \end{pmatrix} \qquad (25)$$

Structural invariants

The closed loop system (25) admits structural invariants of the form

$$\kappa_0 = C(x_e) = \int_a^b \Psi^T x_e d\zeta \qquad (26)$$

with $\Psi = (\psi_1, \psi_2, \psi_3)$ if and only if

$$-\mathcal{G}\psi_2(\zeta) = 0 = -\mathcal{B}_c\psi_2(\zeta) + \mathcal{J}_c^*\psi_3(\zeta) \qquad (27)$$

$$(\mathcal{S}_c + R)\,\psi_2(\zeta) = 0 \qquad (28)$$

$$\mathcal{G}\psi_1(\zeta) + \mathcal{B}_c^*\psi_3(\zeta) = 0 \qquad (29)$$

$$\left.\begin{pmatrix} 0 & G_1 & 0 \\ -G_1^T & 0 & -B_{c1} \\ 0 & B_{c1}^T & J_{c1} \end{pmatrix} \begin{pmatrix} \psi_1(\zeta) \\ \psi_2(\zeta) \\ \psi_3(\zeta) \end{pmatrix}\right|_{a,\,b} = 0 \qquad (30)$$

Control by interconnection : ideal case

• Ideal case : the control acts at each point ζ of the spatial domain.
The controller is of the form

$$\begin{cases} \frac{\partial x_C}{\partial t}(\zeta,t) = \mathcal{J}_c \mathcal{Q}_c x_C(\zeta,t) + \mathcal{B}_c u_C(\zeta,t) \\ y_C(\zeta,t) = \mathcal{B}_c{}^* \mathcal{Q}_c x_C(\zeta,t) + \mathcal{S}_c u_C(\zeta,t) \end{cases} \tag{23}$$

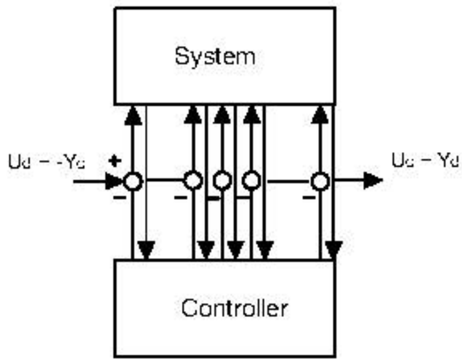
where $\mathcal{Q}_c(\zeta) = \mathcal{Q}_c^T(\zeta)$ and $\mathcal{Q}_c(\zeta) \geq \eta_c$ with $\eta_c > 0$ for all $\zeta \in [a,b]$, $\mathcal{S}_c$ and $\mathcal{S}_c(\zeta) = \mathcal{S}_c^T(\zeta)$ and $\mathcal{S}_c(\zeta) \geq \eta_s$ with $\eta_s > 0$ for all $\zeta \in [a,b]$ and :

$$\mathcal{B}_c = B_{c0} + B_{c1}\frac{\partial}{\partial \zeta}, \text{ and } \mathcal{J}_c = J_{c0} + J_{c1}\frac{\partial}{\partial \zeta} \tag{24}$$

with $B_{c0}, B_{c1} \in \mathbb{R}^{(n_c,1)}, J_{c0} = -J_{c0}^T, J_{c1} = J_{c1}^T \in \mathbb{R}^{(n_c,n_c)}$.

Energy shaping

- In domain control case : we consider now in domain control



and the system is connected to the controller in a power preserving way :

$$\begin{pmatrix} u_d(\zeta,t) \\ y_d(\zeta,t) \end{pmatrix} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \begin{pmatrix} u_C(\zeta,t) \\ y_C(\zeta,t) \end{pmatrix} + \begin{pmatrix} u'(\zeta,t) \\ 0 \end{pmatrix}, \tag{22}$$

Implementation on the elastic string example

We consider here that

- The position of the end point *i.e.* $\omega(b, t)$, is measured .
- The state is reconstructed using a Luenberger PH finite dimensional observer (the controller uses the observer state)$\Rightarrow$ the closed loop stability is guaranteed [Toledo et al., 2020].

Energy shaping

Proposition

Under the hypothesis that the Casimir functions exist, the closed-loop dynamics (when $u = y_c + u'$) is given by :

$$\begin{aligned} \frac{\partial x}{\partial t}(t,\zeta) &= P_1 \frac{\partial}{\partial \zeta} \frac{\delta H_{cl}}{\delta x}(x(t))(\zeta) + (P_0 - G_0)\frac{\delta H_{cl}}{\delta x}(x(t))(\zeta) \\ u' &= W' R \begin{pmatrix} \left(\frac{\delta H_{cl}}{\delta x}(x)\right)(b) \\ \left(\frac{\delta H_{cl}}{\delta x}(x)\right)(a) \end{pmatrix} \end{aligned} \tag{20}$$

in which δ denotes the variational derivative, while

$$H_{cl}(x(t)) = \frac{1}{2}\|x(t)\|_{cl}^2 + \frac{1}{2}\left(\int_a^b \hat{\Psi}^T(\zeta)x(t,\zeta)\,dz\right)^T \times \\ \times \hat{\Gamma}^{-1} Q_C \hat{\Gamma}^{-T} \int_a^b \hat{\Psi}(\zeta)^T x(t,\zeta)\,dz \tag{21}$$

and W' is a $n \times 2n$ full rank, real matrix s.t. $W'\Sigma W'^T \geq 0$.

Energy shaping

- Boundary control case : Asymptotic stabilisation [Macchelli et al., 2017a], Exponential stabilisation [Macchelli et al., 2020] $\Rightarrow$ Control (through $(J_C - R_C, G_C + P_C, (G_C + P_C)^T, M_C + S_C)$) = integrals of the state over the spatial domain.

Casimir functions

Consider the closed loop boundary control system with $u' = 0$ then,

$$C(x(t), x_C(t)) = \Gamma^T x_C(t) + \int_a^b \psi^T(\zeta) x(t,\zeta) dz$$

is a Casimir function for this system if and only if $\psi \in H^1(a, b; \mathbb{R}^n)$,

$$P_1 \frac{d\psi}{dz}(\zeta) + (P_0 + G_0)\psi(\zeta) = 0 \quad (17)$$

$$(J_C + R_C)\Gamma + (G_C + P_C)\tilde{W} R \begin{pmatrix} \psi(b) \\ \psi(a) \end{pmatrix} = 0 \quad (18)$$

$$(G_C - P_C)^T \Gamma + \left[W + (M_C - S_C)\tilde{W} \right] R \begin{pmatrix} \psi(b) \\ \psi(a) \end{pmatrix} = 0 \quad (19)$$

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Energy shaping

Objectives

Modification of the closed loop system's properties (energy shaping) + stabilization (damping injection).

From the power preserving interconnection

$$H_{cl}(x, x_c) = H(x) + H_c(x_c)$$

We first look for structural invariants $C(x, x_c)$ *i.e.* $\frac{dC}{dt} = 0$

$$C(x, x_c) = x_c + F(x) = \kappa$$

where F is a smooth function. In this case the closed loop energy function reads

$$H_{cl}(x, x_c) = H_{cl}(x) = H(x) + H_c(\kappa - F(x))$$

Asymptotic stability of the closed loop system in x^* is achieved using damping injection such that

$$\frac{dH_{cl}}{dt} < 0, \forall x \neq x^*.$$

Energy shaping

Objectives

Modification of the closed loop system's properties (energy shaping) + stabilization (damping injection).

From the power preserving interconnection

$$H_{cl}(x, x_c) = H(x) + H_c(x_c)$$

We first look for structural invariants $C(x, x_c)$ *i.e.* $\frac{dC}{dt} = 0$

$$C(x, x_c) = x_c + F(x) = \kappa$$

where F is a smooth function.

Energy shaping

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From the power preserving interconnection

$$H_{cl}(x, x_c) = H(x) + H_c(x_c)$$

Energy shaping

Objectives

Modification of the closed loop system's properties (energy shaping) + stabilization (damping injection).

Control by interconnection

The system is interconnected with a dynamic controller in a power preserving way.

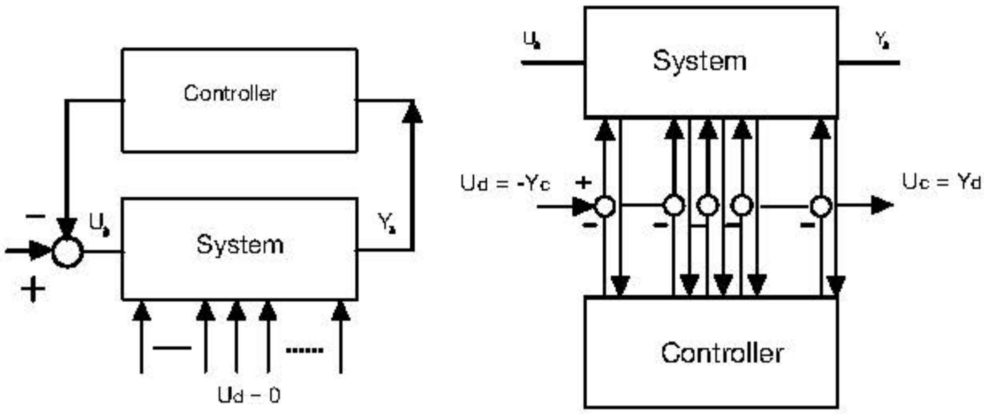


FIGURE – Control by interconnection. Boundary control (left), in domain control (right).

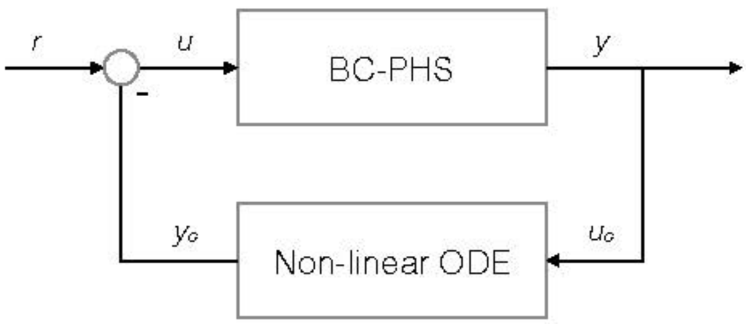
The closed loop energy is equal to the sum of the open loop energy and the controller energy.

Outline

1. Context and motivation
2. Infinite dimensional Port Hamiltonian systems (PHS)
3. Control by interconnection and energy shaping
4. Irreversible boundary controlled port Hamiltonian Systems
5. Conclusions and future works

Non linear case

The previous results have been generalized to the non-linear case [Ramirez et al., 2017] (under some assumptions).



with

$$NL \left\{ \begin{array}{ll} \dot{v}_1 & = K_2 v_2 \\ \dot{v}_2 & = -\frac{\partial \mathcal{P}}{\partial v_1}(v_1)^\top - R(K_2 v_2) + B_c u_c \\ y_c & = B_c^\top K_2 v_2 + S_c u_c \end{array} \right. \tag{16}$$

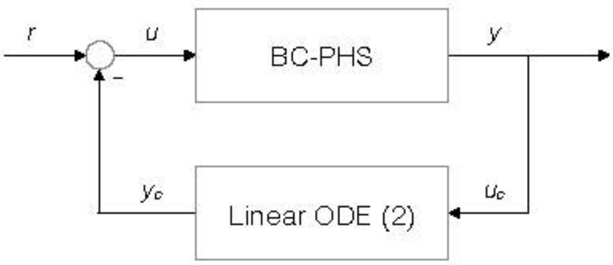
where $v_1 \in \mathbb{R}^{n_c}$, $v_2 \in \mathbb{R}^{n_c}$, form the components of the state vector, $B_c \in M_{k,n_c}(\mathbb{R})$, $K_2 \in M_{n_c}(\mathbb{R})$, $K_2 = K_2^\top$, $K_2 > 0$, $S_c \in M_k(\mathbb{R})$ with $S_c = S_c^\top$ and $S_c \geq 0$.

Dynamic boundary feedback

We consider the controller as linear finite dimensional port Hamiltonian system

$$\dot{v} = (J_c - R_c)\, Q_c v + B_c u_c, \quad y_c = B_c^T Q_c v + S_c u_c, \quad J_c = -J_c^\top, \quad R_c = R_c^\top \geq 0$$

with storage function $E_c(t) = \frac{1}{2}\langle v(t) Q_c v(t)\rangle_{\mathbb{R}^m}$, $Q_c = Q_c^\top > 0 \in \mathbb{R}^m \times \mathbb{R}^m$.



Stability

If the following conditions are satisfied

- $\|u(t)\|^2 + \|y(t)\|^2 \geq \epsilon \|\mathcal{H}x(t,b)\|^2$, $\epsilon > 0$
- power preserving interconnection $u = -y_c + r$, and $u_c = y$
- the controller is assumed to be exponentially stable, i.e., $A_c := (J_c - R_c)Q_c$ is Hurwitz and strictly input passive i.e., $S_c > 0$.

The closed loop system is exponentially stable.

This result has been used for robust tracking control design using internal model principle [Paunonen et al., 2021].

Dynamic boundary feedback

We consider the controller as linear finite dimensional port Hamiltonian system

$$\dot{v} = (J_c - R_c)\, Q_c v + B_c u_c, \quad y_c = B_c^T Q_c v, \quad J_c = -J_c^\top, \quad R_c = R_c^\top \geq 0$$

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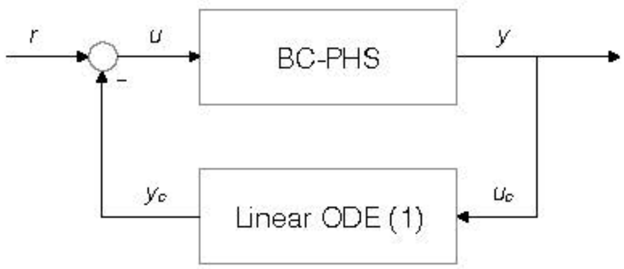

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- power preserving interconnection $u = -y_c + r$, and $u_c = y$
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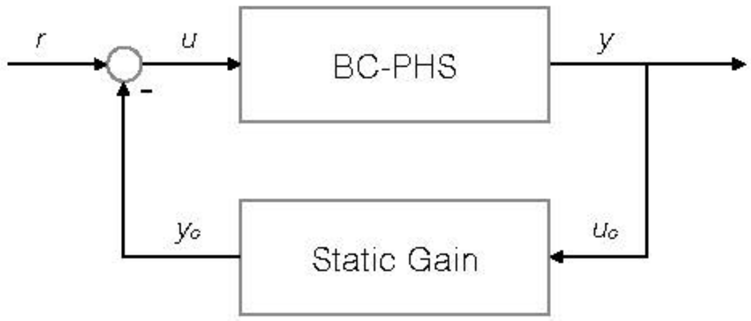
The closed loop system is asymptotically stable.

Static feedback control

Impedance passive case

In the impedance passive case the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{H}} \leq u^{\top}(t)y(t).$$



Static controller : α

- Asymptotic stability :
  $\alpha > 0$+(compactness condition)
- Exponential stability
  [Villegas et al., 2009] : α st

$$(dE/dt) \leq -k\|(\mathcal{H}x)(t,b)\|^2_{\mathbb{R}}$$

where $k > 0$.

This result has been used for observer design [Toledo et al., 2020].

Static feedback control

Impedance passive case

In the impedance passive case the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{H}} \leq u^\top(t)y(t).$$

The vibrating string example

The boundary port variables are

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v(b) - v(a) \\ \sigma(b) - \sigma(a) \\ \sigma(b) + \sigma(a) \\ v(b) + v(a) \end{pmatrix}$$

The boundary input and output are selected as

$$u(t) = \begin{pmatrix} v(a,t) \\ \sigma(b,t) \end{pmatrix} \qquad y(t) = \begin{pmatrix} -\sigma(a,t) \\ v(b,t) \end{pmatrix} \tag{15}$$

which can be derived choosing W and $\tilde{W}$ such that :

$$W = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad \tilde{W} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

The energy balance is then :

$$\frac{\mathrm{d}H}{\mathrm{d}t}(t) \leq y^{\mathrm{T}}(t)u(t).$$

The vibrating string example

The vibrating string equation is given by

$$\frac{\partial^2 \omega(\zeta,t)}{\partial t^2} = \frac{1}{\mu(\zeta)} \frac{\partial}{\partial \zeta}\left(T(\zeta)\frac{\partial \omega(\zeta,t)}{\partial \zeta}\right) - D\frac{\partial \omega(\zeta,t)}{\partial t}$$

and can be recasted in a PHS form choosing $\varepsilon = \frac{\partial \omega(\zeta,t)}{\partial \zeta}$ and $p = \mu \frac{\partial \omega(\zeta,t)}{\partial t}$ as state variables.

$$\frac{\partial}{\partial t}\begin{pmatrix} \varepsilon \\ p \end{pmatrix} = \begin{pmatrix} 0 & \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial \zeta} & -D \end{pmatrix}\begin{pmatrix} T & 0 \\ 0 & \frac{1}{\mu} \end{pmatrix}\begin{pmatrix} \varepsilon \\ p \end{pmatrix}$$

which is on the form

$$\frac{\partial x}{\partial t}(\zeta,t) = \left(P_1 \frac{\partial}{\partial \zeta} + P_0 - R_0\right)[\mathcal{H}x(\zeta,t)]$$

with

$$P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, R_0 = \begin{pmatrix} 0 & 0 \\ 0 & D \end{pmatrix}, \mathcal{H} = \begin{pmatrix} T & 0 \\ 0 & \frac{1}{\mu} \end{pmatrix}, \mathcal{H}x(\zeta,t) = \begin{pmatrix} \sigma(\zeta,t) \\ v(\zeta,t) \end{pmatrix}$$

Boundary controlled port Hamiltonian systems

The general formulation (1) allows to model a large class of systems.

For example :

- The 1D wave equation where $n = 1$, $N = 1$, $G_0 = 0, G_1 = 1$.
- The Euler Bernouilli beam equation. In this case $n = 1$, $N = 2$, $G_0 = 0$, $G_1 = 0, G_2 = 1$.
- The Timoshenko beam equation. In this case $n = 2$, $N = 1$, and

$$G_0 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In what follows we focus on first order differential operators

Boundary controlled port Hamiltonian systems

Existence of solution [Le Gorrec et al., 2005]

The operator

$$\mathcal{J} = \sum_{i=0}^{N} P_i \frac{\partial^i}{\partial \zeta^i} \left(\mathcal{H}(\zeta)x(\zeta,t)\right) - R_0 \mathcal{H}(\zeta)x(\zeta,t)$$

with domain

$$D(\mathcal{J}) = \left\{ \mathcal{H} \in H^N\left(a,b;\mathbb{R}^n\right) \mid \begin{bmatrix} \mathcal{H}(b)x(b,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial \zeta^{N-1}}(b,t) \\ \mathcal{H}(a)x(a,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial \zeta^{N-1}}(a,t) \end{bmatrix} \in KerW_B \right\}$$

where W_B is defined by (9) and Ξ_1 and Ξ_2 satisfy (12), generates a contraction semigroup on X. Furthermore the system (5-7) with (9-10) and (12) defines a boundary control system.

Boundary controlled port Hamiltonian systems

where

$$P_e = \begin{bmatrix} P_1 & \cdots & (-1)^{N-1} P_N \\ \vdots & \ddots & 0 \\ (-1)^{N-1} P_N & 0 & 0 \end{bmatrix} \tag{11}$$

and Ξ_1 and Ξ_2 in $\mathbb{R}^{k \times k}$ satisfy

$$\Xi_2^\top \Xi_1 + \Xi_1^\top \Xi_2 = 0, \text{ and } \Xi_2^\top \Xi_2 + \Xi_1^\top \Xi_1 = I \tag{12}$$

The energy balance associated to the system reads

$$\frac{dH}{dt} = \int_a^b y_d^T u_d \, d\zeta - \int_a^b \left(x_2^T(\zeta, t) \mathcal{H}_2^T(\zeta) R \mathcal{H}_2(\zeta) x_2(\zeta, t) \right) d\zeta + y_\partial^T u_\partial \tag{13}$$

$$\leq \int_a^b y_d^T u_d \, d\zeta + y_\partial^T u_\partial \tag{14}$$

Boundary controlled port Hamiltonian systems

Mixed in-domain / boundary controlled port Hamiltonian systems (IDBC-PHS)

A mixed in-domain / boundary controlled port Hamiltonian system is an infinite dimensional system of the form (5-7) where

$$u_\partial = W_B \begin{bmatrix} \mathcal{H}(b)x(b,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial\zeta^{N-1}}(b,t) \\ \mathcal{H}(a)x(a,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial\zeta^{N-1}}(a,t) \end{bmatrix}, \text{ and } y_\partial = W_C \begin{bmatrix} \mathcal{H}(b)x(b,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial\zeta^{N-1}}(b,t) \\ \mathcal{H}(a)x(a,t) \\ \vdots \\ \frac{\partial^{N-1}(\mathcal{H}x)}{\partial\zeta^{N-1}}(a,t) \end{bmatrix} \quad (8)$$

with

$$W_B = \left[\tfrac{1}{\sqrt{2}} (\Xi_2 + \Xi_1 P_e) \quad \tfrac{1}{\sqrt{2}} (\Xi_2 - \Xi_1 P_e) \right], \quad (9)$$

$$W_C = \left[\tfrac{1}{\sqrt{2}} (\Xi_1 + \Xi_2 P_e) \quad \tfrac{1}{\sqrt{2}} (\Xi_1 - \Xi_2 P_e) \right], \quad (10)$$

Infinite dimensional Port Hamiltonian systems (PHS)

For a sake of compactness we shall use the following notation

$$P_i = \begin{bmatrix} 0 & G_i \\ (-1)^{i+1} G_i^T & 0 \end{bmatrix}, \; R_0 = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} \tag{4}$$

and the formulation of (1)

$$\frac{\partial x}{\partial t}(\zeta, t) = \sum_{i=0}^{N} P_i \frac{\partial^i}{\partial \zeta^i} \left(\mathcal{H}(\zeta) x(\zeta, t)\right) - R_0 \mathcal{H}(\zeta) x(\zeta, t) + \begin{bmatrix} 0 \\ I \end{bmatrix} u_d(\zeta, t) \tag{5}$$

$$y_d(\zeta, t) = \begin{bmatrix} 0 & I \end{bmatrix} \mathcal{H}(\zeta) x(\zeta, t) \tag{6}$$

$$u_\partial = \mathcal{B}\left(\mathcal{H}(\zeta) x(\zeta, t)\right), y_\partial = \mathcal{C}\left(\mathcal{H}(\zeta) x(\zeta, t)\right) \tag{7}$$

The total energy of the system $H(x)$ is defined by

$$H(x) = \frac{1}{2} \int_a^b \left(x^T(\zeta, t) \mathcal{H}(\zeta) x(\zeta, t)\right) d\zeta$$

Infinite dimensional Port Hamiltonian systems (PHS)

Infinite dimensional Port Hamiltonian systems (PHS)

$$\frac{\partial}{\partial t}\begin{bmatrix} x_1(\zeta,t) \\ x_2(\zeta,t)\end{bmatrix} = \begin{bmatrix} 0 & \mathcal{G} \\ -\mathcal{G}^* & -R\end{bmatrix}\begin{bmatrix} \mathcal{H}_1(\zeta)x_1(\zeta,t) \\ \mathcal{H}_2(\zeta)x_2(\zeta,t)\end{bmatrix} + \begin{bmatrix} 0 \\ I\end{bmatrix} u_d(\zeta,t) \quad (1)$$

$$y_d(\zeta,t) = \begin{bmatrix} 0 & I\end{bmatrix}\begin{bmatrix} \mathcal{H}_1(\zeta)x_1(\zeta,t) \\ \mathcal{H}_2(\zeta)x_2(\zeta,t)\end{bmatrix} \quad (2)$$

$$u_\partial = \mathcal{B}\begin{bmatrix} \mathcal{H}_1(\zeta)x_1(\zeta,t) \\ \mathcal{H}_2(\zeta)x_2(\zeta,t)\end{bmatrix}, \; y_\partial = \mathcal{C}\begin{bmatrix} \mathcal{H}_1(\zeta)x_1(\zeta,t) \\ \mathcal{H}_2(\zeta)x_2(\zeta,t)\end{bmatrix} \quad (3)$$

where $x = [x_1^T, x_2^T]^T \in X := L^2([a,b], \mathbb{R}^n) \times L^2([a,b], \mathbb{R}^n)$, $\mathcal{H} = diag(\mathcal{H}_1, \mathcal{H}_2)$ and $\mathcal{H}(\zeta) = \mathcal{H}^T(\zeta)$ and $\mathcal{H}(\zeta) \geq \eta$ with $\eta > 0$ for all $\zeta \in [a,b]$, $R \in \mathbb{R}^{(n,n)}$, $R = R^T > 0$, $\mathcal{B}(\cdot)$ and $\mathcal{C}(\cdot)$ are some boundary input and boundary output mapping operators. Furthermore

$$\mathcal{G} = \sum_{i=0}^{N} G_i \frac{\partial^i}{\partial \zeta^i}, \text{ and } \mathcal{G}^* = \sum_{i=0}^{N} (-1)^i G_i^T \frac{\partial^i}{\partial \zeta^i}$$

with $G_i \in \mathbb{R}^{(n,n)}$.

Outline

1. Context and motivation
2. Infinite dimensional Port Hamiltonian systems (PHS)
3. Control by interconnection and energy shaping
4. Irreversible boundary controlled port Hamiltonian Systems
5. Conclusions and future works

Context : port Hamiltonian systems

In the linear 1D case this formalism has been used for

- Proving existence of solution using the semi-group theory [Le Gorrec et al., 2005].
- Stability analysis (when interconnected with linear or non linear ODEs) : asymptotic or exponential [Ramirez et al., 2017, Augner, 2016].
- Simulation through structure preserving schemes [Trenchant et al., 2018, Kotyczka et al., 2019].
- Control design : control by interconnection, energy shaping, observer design, backstepping ... [Macchelli et al., 2017b, Toledo et al., 2020, Redaud et al., 2022].

Some extensions have been proposed for

- Multidimensional systems [Skrepek, 2021].
- Implicit systems [Heidari and Zwart, 2022].
- Non linear PDE systems such as 1D or 2D-3D fluids ([Mora et al., 2021]) using Irreversible port Hamiltonian Formulations ([Ramirez et al., 2022]).

In this talk we recall some well known results on boundary controlled port Hamiltonian systems and consider energy shaping using boundary or in domain control. Extension to IPHS.

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Context : port Hamiltonian systems

Port Hamiltonian systems (PHS)

Class of non linear dynamic systems derived from an extension to open physical systems (1992) of Hamiltonian and Gradient systems. This class has been generalized (2001) to distributed parameter systems.

$$x(t) : \begin{cases} \dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + B(x)u \\ y = B(x)^T \frac{\partial H(x)}{\partial x} \\ \frac{dH}{dt} \leq y^T u \end{cases} \qquad x(t,\zeta) : \begin{cases} \dot{x} = (\mathcal{J}(x) - \mathcal{R}(x))\frac{\delta H(x)}{\delta x} + \mathcal{B}_d u_d \\ y_d = \mathcal{B}_d^* \frac{\delta H(x)}{\delta x} \\ \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{\delta H(x)}{\delta x}|_\partial, \\ \frac{dH}{dt} \leq y_d^T u_d + f_\partial^T e_\partial \end{cases},$$

- Port Hamiltonian systems :
  - The state variables are chosen as the energy variables.
  - The links between the energy function and the system dynamics is made explicit through symmetries.
  - The boundary port variables are power conjugated.
- "Easy" to extend to non linear or systems defined on higher dimensional spaces.

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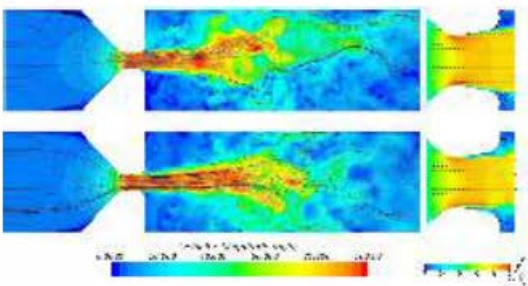
Context / Motivation : Toward complex systems and structures

- Soft robotics (FEMTO-ST France)


- Fluid systems
  - Modeling and control of interglotal air flows (coll. USM Chile)
  - Artificial aorta for blood pressure control (coll. EPFL Swizerland)



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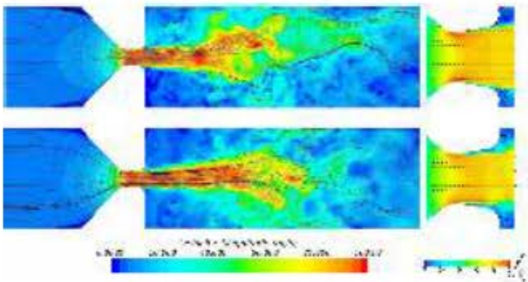

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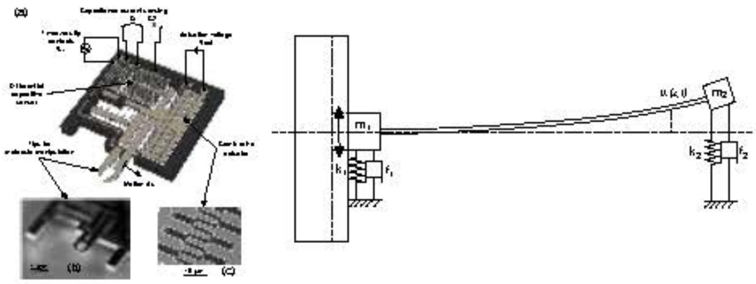
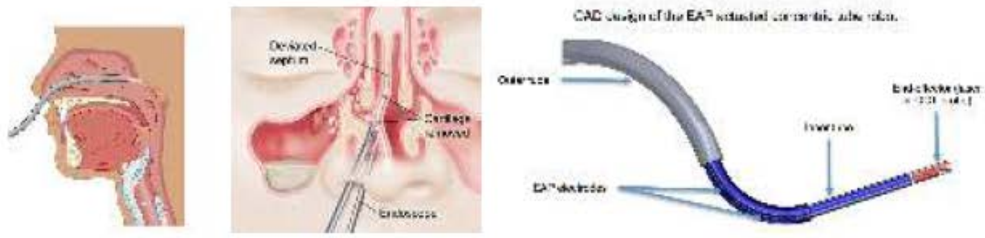
Context / Motivation : control of flexible structures

- Boundary controlled systems (e.g. Control of nanotweezers - Coll. LIMMS, Tokyo)



- In-domain control of distributed parameter systems (e.g. Control of smart endoscopes, FEMTO-ST)



  - Exploration, imaging, diagnosis.
  - Mini invasive surgery.
  - Toward miniaturized and *smart* endoscopes.

Context / Motivation : control of flexible structures

- Boundary controlled systems (e.g. Control of nanotweezers - Coll. LIMMS, Tokyo)